Solving Linear Equations

1. Solve the following equations:

(a)	$7x+1=15$	(b)	$3x+5=29$	(c)	$4x+1=37$
(d)	$11x-3=19$	(e)	$4x-7=13$	(f)	$5x-11=-1$
(g)	$6x-13=-7$	(h)	$3-4x=-17$	(i)	$11-8x=-5$
(j)	$9-4x=-19$	(k)	$14-3x=-1$	(l)	$17-5x=32$

2. Solve the following equations:

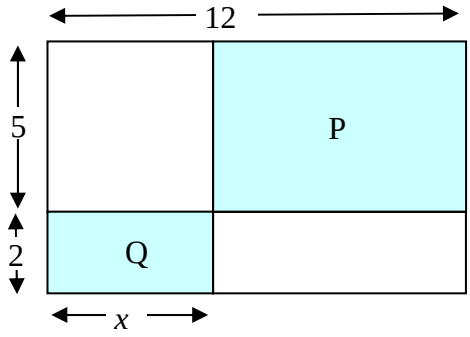
(a)	$7x+3=5x+7$	(b)	$11x-7=8x+2$
(c)	$12x+11=5x-3$	(d)	$9x-1=7x+7$
(e)	$7+4x=6x-3$	(f)	$11-x=3x+19$
(g)	$13-x=x+11$	(h)	$13-5x=28-2x$
(i)	$17x-2=18-3x$	(j)	$17-8x=29-14x$

3. The sum of four consecutive numbers is 98. Let the first number be x and write down the other three numbers in terms of x. Find the four numbers.

4. The sum of four consecutive *odd* numbers is 216. Find the numbers.

5. You have three consecutive even numbers so that the sum of twice the smallest number plus three times the middle number is four times the largest number. Find the three numbers.

6.
(a) The area of rectangle P is five times the area of rectangle Q. Find x.
(b) The value of x is changed and the areas of the unshaded rectangles become equal. Find the new value of x.

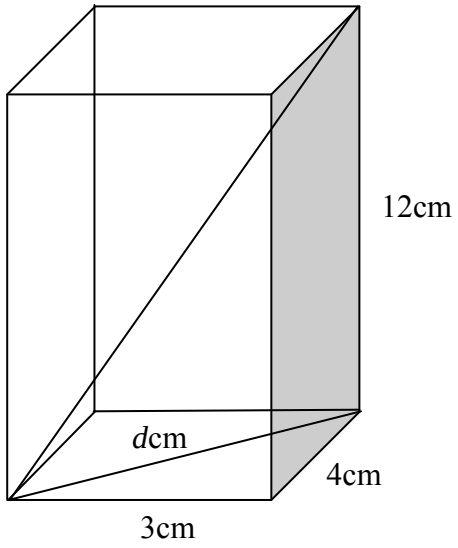


Pythagoras' Theorem without a Calculator

(You must draw a clearly labelled triangle for each question)

1. (a) A rectangle measures 3cm by 4cm. Find the length of the diagonal.
   (b) The hypotenuse of a right-angled triangle is 13cm and its shortest side is 5cm. Find the length of the third side.

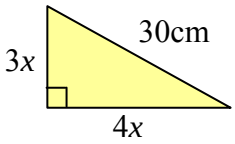
2. A box with width 3cm length 4cm and height 12cm. Is shown below:



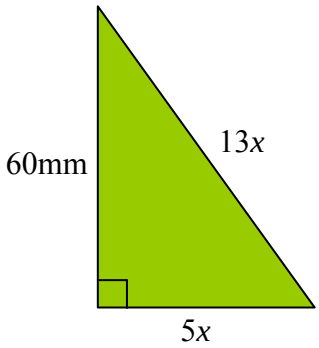
   (a) What is the length of the diagonal, d, of the base of the box?
   (b) What is the length of the longest pencil which can be fitted into the box? (Assume the pencil is infinitely thin).

3. Find x in the following diagrams:

(a)



(b)



4. A field measures 70m by 240m. What is the length of the diagonal of the field?

5. Find the distance between the following points:
   (a) (7, 9) and (11, 12)
   (b) (-2, 3) and (4, -5)
   (c) (-1, -2) and (-6, 10)
   (d) (1, 14) and (9, -1)

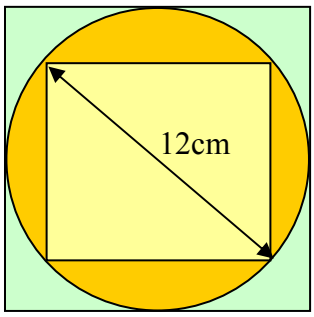
PTO

6. Find the third side in a right-angled triangle whose hypotenuse is 20cm and whose shortest side is 12cm.

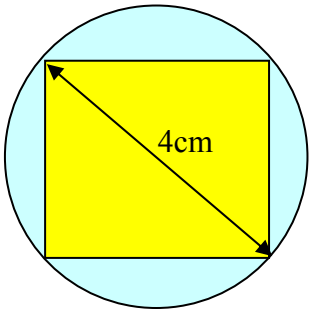
7. Find the hypotenuse of the right-angled triangle whose shortest two sides are 20cm and 48cm.

8. Water is poured into a hemispherical bowl of radius 30cm so that it has a depth of 12cm. Find the radius of the water in the bowl.

9. A square is drawn inside a circle as shown above whose diameter is 12cm. The circle touches the larger square at four points as shown and the smaller square has its four corners on the circle.
   (a) If the side length of the smaller square is x then write down an equation involving x.
   (b) Hence show that the area of the smaller square is 72 cm^2.
   (c) Use this to find the value of n in the following equation :
   $\text{Area of larger square} = n \times (\text{Area of smaller square})$



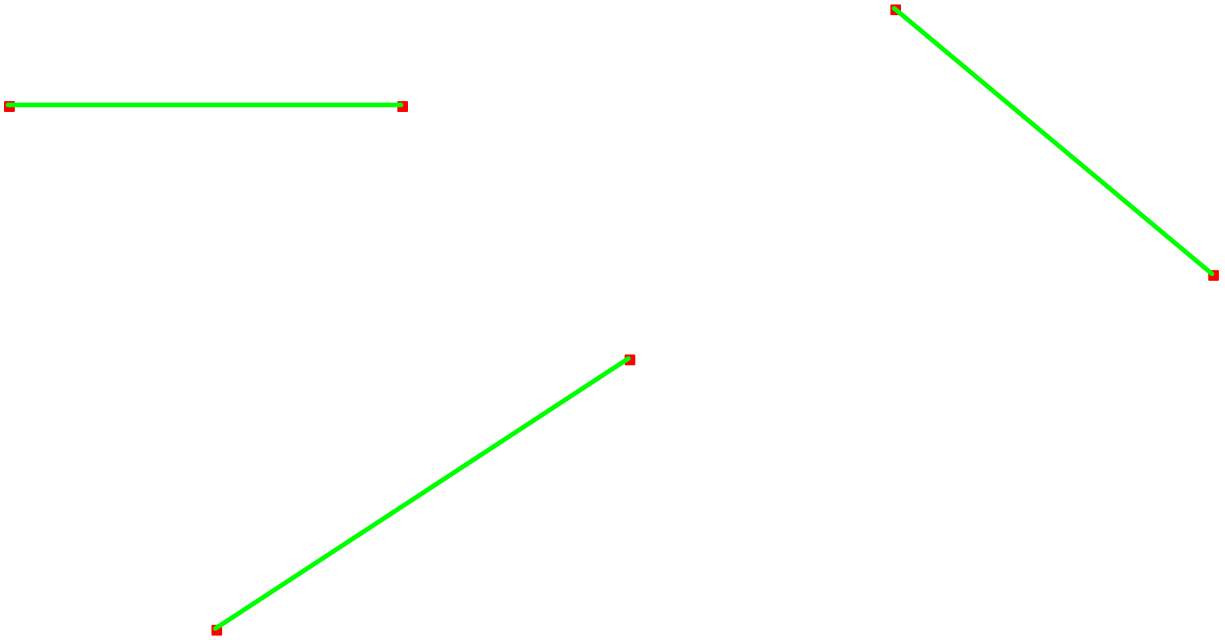
10. A square is drawn inside a circle as shown below so that its four corners lie on the square.
   (a) If the diameter of the circle is 4cm then find the side length of the square, leaving your answer in the form $\sqrt{n}$.
   (b) Hence write down the exact area of the square.
   (c) Use this to find the area between the circle and the square (in terms of π).



Constructions using a Straight Edge and a Compass

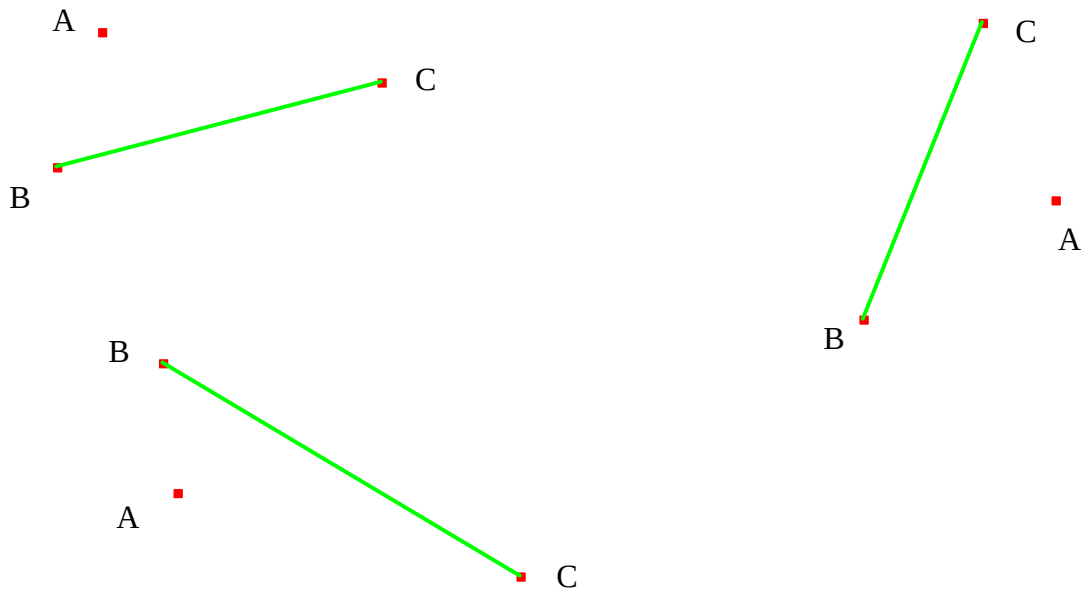
In the following use a compass and ruler, showing your working clearly.

1. Construct the perpendicular bisectors of the following solid lines.



2. Check with a ruler whether you have cut the lines in half.

3. Construct the lines which are perpendicular to the line *BC* and which pass through the point *A*.



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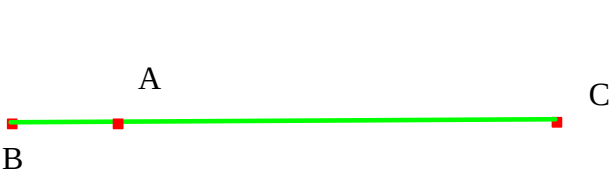
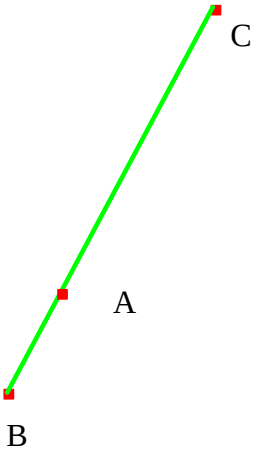
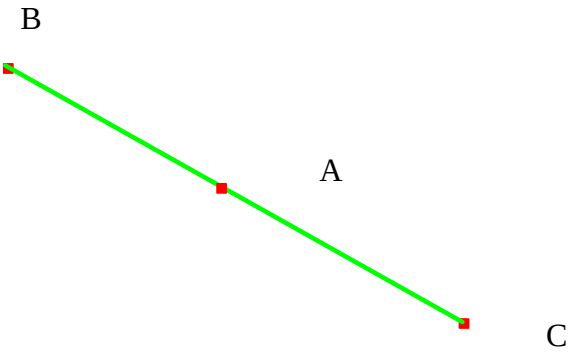
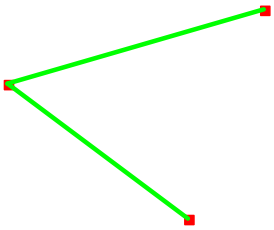
4. Construct the lines which are perpendicular to the line *BC* and which pass through the point *A*.

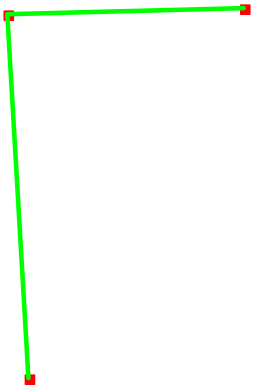


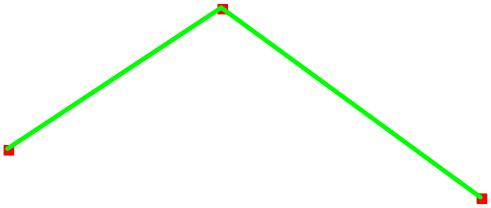




5. Construct the lines which bisect the angles enclosed between these pairs of lines:







Indices

1. Simplify the following:

(a) $a^2 \times a^3$ (b) $b^3 \times b^{14}$

(c) $c^6 \times c^5$ (d) $x^6 \times x^7$

(e) $y^9 \div y^2$ (f) $z^{11} \div z^4$

(g) $\left(q^3\right)^4$ (h) $\left(w^2\right)^6$

(i) $\left(r^2\right)^3 \div r^4$ (j) $\left(s^5\right)^3 \times s^6$

(k) $\dfrac{t^2 \times t^5}{t^3}$ (l) $\dfrac{\left(w^3\right)^4 \times w^6}{w^2}$

2. Find a in the following:

(a) $2^3 \times 2^a = 2^7$ (b) $x^5 \times x^a = x^6$ (c) $y^2 \times y^a = y^2$

(d) $r^a \div r^{11} = r^9$ (e) $s^a \div s^9 = s^7$ (f) $2^9 \div 2^a = 2^2$

(g) $\left(3^5\right)^a = 3^{20}$ (h) $\left(u^a\right)^a = u^9$ (i) $\left(x^a\right)^{a+1} = x^{42}$

3. Simplify the following:

(a) $\left(b^2\right)^3 \times \left(b^4\right)^5$ (b) $\dfrac{\left(t^3\right)^5 \times \left(t^2\right)^4}{t^3}$

(c) $\dfrac{\left(g^2\right)^3 \times \left(g^3\right)^2}{\left(g^5\right)^2}$ (d) $\dfrac{\left(h^3\right)^4 \times \left(h^2\right)^7}{\left(h^5\right)^2 \times \left(h^3\right)^4}$

(e) $\dfrac{\left(k^5\right)^7 \times \left(k^3\right)^4}{\left(k^8\right)^3 \times \left(k^2\right)^2}$ (f) $\dfrac{\left(m^2\right)^9 \times \left(m^7\right)^3}{\left(m^4\right)^3 \times \left(m^2\right)^{11}}$

4. Simplify the following:

(a) $\left(2x^2\right) \times \left(3x^4\right)$ (b) $\left(5x^3\right) \times \left(7x^2\right)$

(c) $\left(6x^6\right) \times \left(2x^3\right)$ (d) $\left(3x^5\right) \times \left(9x^3\right)$

(e) $\left(8x^2\right) \times \left(6x^5\right)$ (f) $\left(7x^7\right) \times \left(8x^5\right)$

(g) $\left(9x^7\right) \times \left(4x^3\right)$ (h) $\left(9x^2\right) \times \left(7x\right)$

5. Simplify the following:

(a) $\left(8x^5\right) \div \left(2x^3\right)$ (b) $\left(6x^4\right) \div \left(3x\right)$

(c) $\left(24x^6\right) \div \left(6x^4\right)$ (d) $\left(12x^7\right) \div \left(4x^3\right)$

(e) $\left(9x^5\right) \div \left(3x^2\right)$ (f) $\left(63x^8\right) \div \left(9x^7\right)$

(g) $\left(72x^{10}\right) \div \left(8x\right)$ (h) $\left(56x^7\right) \div \left(7x^5\right)$

Simultaneous Equations

1. Solve the following simultaneous equations (by the most efficient method):

(a) $3v + 7w = 32$
$5v = 3w + 2$

(b) $3p + q = 11$
$7q = 32 - 11p$

2. A shops sells two types of pens, one type costs £5 and the other costs £7. One day it sells 17 pens and received £109. By first writing down simultaneous equations find how many of each pen it sold.

3. In a sale a bookshop was selling all its hard backs at the same price and all its paper backs at the same price. A woman bought 7 hard backs and 5 paper backs which cost her £61.40. A man bought 11 hard backs and 7 paper backs which cost him £93.10. Find the price of a hard back and the price of a paper back in the sale (by solving the relevant simultaneous equations).

4. A mother is six times older than her daughter. Let the mother's age be m and the daughter's age be d.
(a) Write down an equation involving m and d.
(b) Write down an expression for the mother's age in two years time.
(c) Write down an expression for the daughter's age in two years time.
In two years time, the mother is five times older than her daughter.
(d) Write down a second equation involving m and d.
(e) Solve the equations of (a) and (d) to find the present age of the mother and daughter.

5. An airline sold twice as many second class tickets as it sold first class tickets for a certain flight. The second class tickets cost £125 and the first class tickets cost £220. The total cost of the tickets was £17,390.
Let f represent the number of first class tickets and s represent the number of second class tickets.
(a) Write down two equations involving f and s.
(b) Solve these equations to find f and s.

6. Solve the following simultaneous equations:

(a) $5x + 3y = 5$
$7x + 2y = 18$

(b) $3p - 2q = 3$
$11p + 5q = 122$

(c) $b = 3a + 1$
$3a + 7b = 79$

(d) $2s = 5r - 3$
$9r + 4s = 89$

7. A man buys three second class and seven first class tickets for a flight which costs him £1625. Another man buys two second class and five first class tickets which costs him £1150.
(a) If the cost of a first class ticket is x and the cost of a second class ticket is y then write down two simultaneous equations involving x and y.
(b) Solve these to find the cost of the first and second class tickets.

Sequences

In all questions t_n refers to the *n*th term of the sequence.

1. Find the next two terms in the following sequence

(a)	5, 7, 9, 11...	(b)	1, 11, 21, 31...	(c)	15, 22, 29, 36...
(d)	2, 11, 20, 29...	(e)	14, 21, 28, 35...	(f)	1, 1½, 2, 2½...
(g)	15, 12, 9, 6...	(h)	101, 99, 97, 95...	(i)	17, 11, 5, –1...
(j)	–3, –7, –11, –15...	(k)	–9, –2, 5, 12...	(l)	–3, –1½, 0, 1½,...
(m)	½, ¾, 1, 1¼,...	(n)	1, ⅞, ¾, ⅝...	(o)	½, 1¼, 2, 2¾,...

2. Find the *n*th term, t_n, of the sequences in question 1.

3. (a) Find the *n*th term, t_n of $5, 11, 17, 23,...$.
   (b) Find the 40^{th} term of $5, 11, 17, 23,...$.
   (c) Which term of $5, 11, 17, 23,...$ is equal to 479?

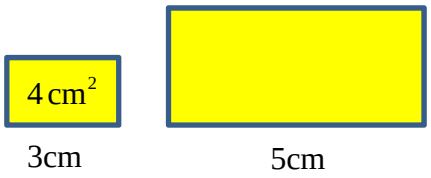
4. (a) Find the *n*th term, t_n of $3, 10, 17, 24,...$
   (b) Find the 71^{st} term of $3, 10, 17, 24,...$.
   (c) Which term of $3, 10, 17, 24,...$ is equal to 710?

5. (a) Find the *n*th term, t_n of $19, 14, 9, 4,...$.
   (b) Find the 23^{rd} term of $19, 14, 9, 4,...$.
   (c) Which term of $19, 14, 9, 4,...$ is equal to –346?

6. (a) Find the *n*th term, t_n of $-3, -5, -7, -9,...$
   (b) Find the 15^{th} term of $-3, -5, -7, -9,...$.
   (c) Which term of $-3, -5, -7, -9,...$ is equal to -287?

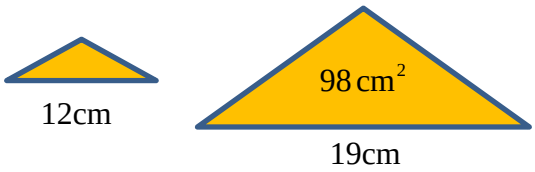
Similar Shapes

1. The two rectangles shown below are similar.



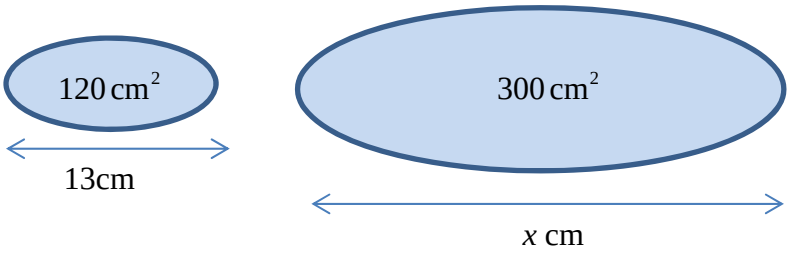
Find (to 3sf) the area of the larger rectangle.

2. The two triangles shown below are similar.



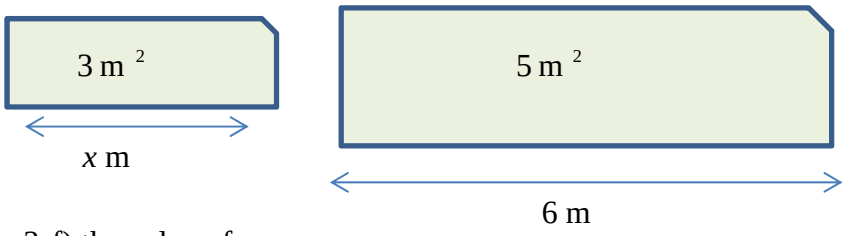
Find (to 3sf) the area of the smaller triangle.

3. The two shapes shown below are similar.



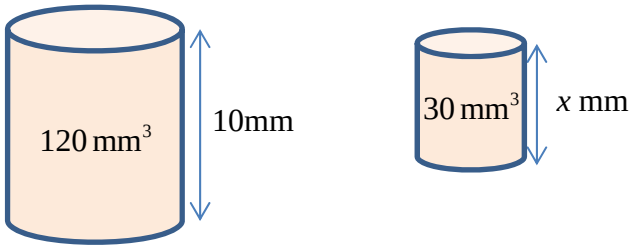
Find (to 3sf) the value of *x*.

4. The two shapes shown below are similar.



Find (to 3sf) the value of *x*.

5. The two shapes shown below are similar.



Find (to 3sf) the value of *x*.

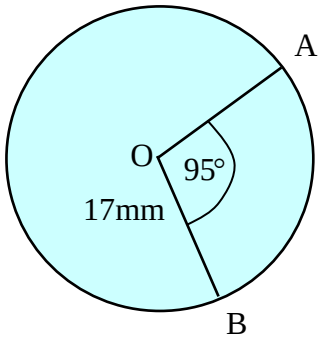
PTO

6. Two bottles of lemonade are similar. One has volume of 1litre and has a height of 25cm. The other has a volume of 1.5 litres. Find its height (to 3sf).

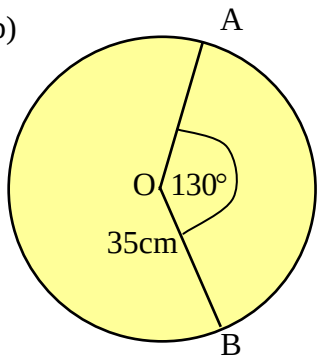
Sectors of Circles

1. Find the area of the sectors OAB (in which the angle is marked) in the diagrams below (to 3sf):

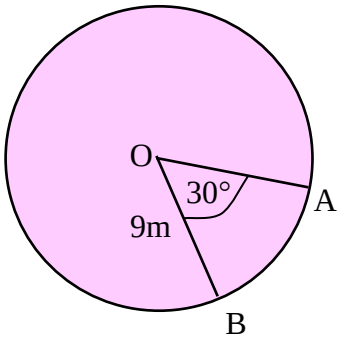
(a)



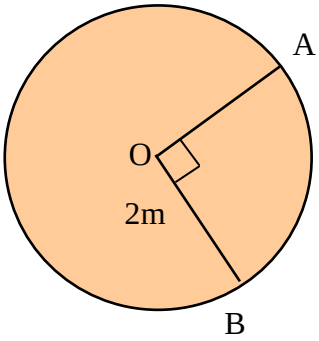
(b)



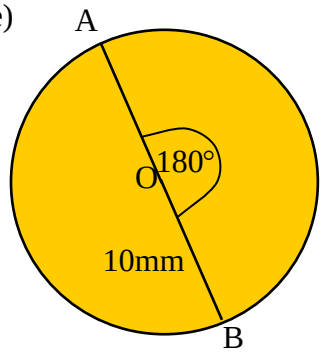
(c)



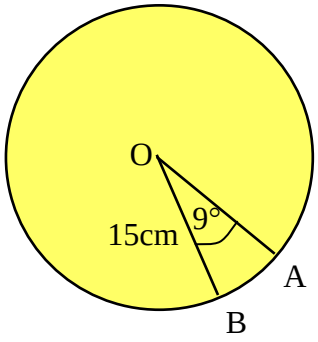
(d)



(e)

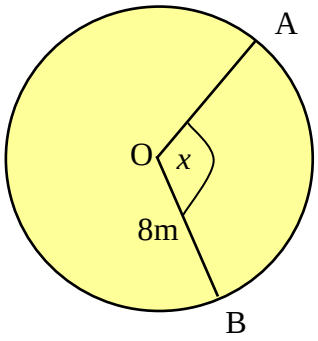


(f)



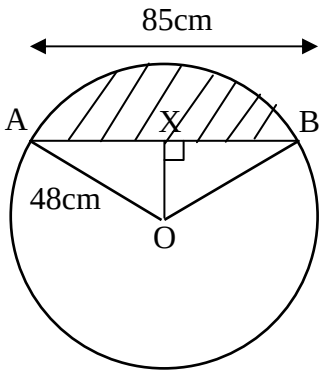
2. Find the length of the curved edge AB in the above diagrams (to 3sf) (travelling clockwise from A to B in every case).

3. The area of the minor sector AOB shown below is 60m^2.
   (a) Find an expression for the area of the minor sector AOB in terms of x.
   (b) By putting this expression equal to 60m^2, find the angle x (to 1dp).
   (c) Find an expression for the length of the minor arc AOB in terms of x.
   (d) Find the *exact* length of the minor arc AB (by using "Ans" on calculator for the value of x).



PTO

4. In the diagram below AB = 85cm, X is the midpoint of AB, and O is the centre of the circle
   (a) Show that $\angle AOX = 62.3°$ (to 1dp) (by using trigonometry).
   (b) Find the angle AOB (to 1dp).
   (c) Find the length OX (to 3sf).
   (d) Find the area of triangle AOB (to 3sf).
   (e) Find the area of the sector AOB (to 3sf).
   (f) Find the area of the shaded segment (to 3sf).



Lowest Common Multiples, Highest Common Factors

1. Write the following as the product of prime numbers (e.g. $72 = 2^3 \times 3^2$)

(a)	30	(b)	24
(c)	18	(d)	28
(e)	105	(f)	64
(g)	108	(h)	42
(i)	150	(j)	200
(k)	378	(l)	144
(m)	385	(n)	126
(o)	420	(p)	45
(q)	60	(r)	75

2. Use question 1 to find the highest common factor of the following pairs of numbers (write each answer first as the product of prime numbers and then calculate this):
   (a) 45 and 105
   (b) 28 and 64
   (c) 385 and 75
   (d) 420 and 126
   (e) 108 and 150

3. Use question 1 to find the lowest common multiple of the following pairs of numbers(write each answer first as the product of prime numbers and then calculate this):
   (a) 24 and 60
   (b) 200 and 420
   (c) 30 and 144
   (d) 42 and 28
   (e) 378 and 18

4. Two cars complete laps of a circuit. One takes 315 seconds per lap, the other takes 525 seconds per lap. They start their circuits of the laps at the same time.

   (a) Express 315 and 525 as the product of prime numbers.
   (b) Find the lowest common multiple of 315 and 525.
   (c) Use this to find how many laps the faster car will do before the cars first get to the starting point at the same time.

5. (a) Find the highest common factor and lowest common multiple of 210 and 550.
   (b) Multiply the two numbers that you found in (a) together.
   (c) Multiply 210 and 550 together.
   (d) What do you notice about the answers to (b) and (c)?

Positive and Negative Indices (without calculators)

1. Express the following as powers of 2 (i.e. in the form 2^n):

(a) 4 (b) 16 (c) 64

(d) 2 (e) $\frac{1}{2}$ (f) 0.25

(g) 1 (h) $\frac{1}{32}$ (i) 128

2. Express the following as powers of the stated numbers:

(a) 32 as a power of 2 (b) 81 as a power of 3

(c) 625 as a power of 5 (d) $\frac{1}{16}$ as a power of 4

(e) $\frac{1}{7}$ as a power of 7 (f) $\frac{1}{25}$ as a power of 5

(g) $\frac{1}{144}$ as a power of 12 (h) $\frac{1}{1024}$ as a power of 2

3. Calculate the following:

(a) 2^6 (b) 3^4 (c) 5^3

(d) 11^2 (e) 2^{-3} (f) 10^{-2}

(g) 19^0 (h) 13^2 (i) 4^{-3}

(j) $\left(\frac{1}{2}\right)^3$ (k) $\left(\frac{2}{3}\right)^2$ (l) $\left(\frac{5}{3}\right)^4$

(m) $\left(\frac{1}{2}\right)^{-2}$ (n) $\left(\frac{2}{5}\right)^{-3}$ (o) $\left(\frac{2}{3}\right)^{-4}$

4. Find x in the following:

(a) $7^x = 49$ (b) $3^x = \frac{1}{81}$ (c) $2^x = 1$

(d) $9^x = 81$ (e) $5^x = \frac{1}{125}$ (f) $\left(\frac{2}{3}\right)^x = \frac{27}{8}$

(g) $\left(\frac{1}{4}\right)^x = 16$ (h) $\left(\frac{3}{4}\right)^x = \frac{16}{9}$ (i) $2^x = 1024$

5. Express the following in the form 8^n where n is either an integer or a fraction:

(a) $\frac{1}{8}$ (b) 8 (c) 1

(d) 64 (e) $\frac{1}{64}$ (f) $\frac{1}{512}$

Simplifying Square Roots

1. Simplify the following as far as possible:

(a) $\sqrt{18}$ (b) $\sqrt{8}$ (c) $\sqrt{12}$

(d) $\sqrt{50}$ (e) $\sqrt{45}$ (f) $\sqrt{44}$

(g) $\sqrt{75}$ (h) $\sqrt{63}$ (i) $\sqrt{72}$

2. Simplify the following as far as possible:

(a) $\sqrt{320}$ (b) $\sqrt{180}$ (c) $\sqrt{300}$

(d) $\sqrt{245}$ (e) $\sqrt{200}$ (f) $\sqrt{343}$

(g) $\sqrt{135}$ (h) $\sqrt{150}$ (i) $\sqrt{216}$

3. Find the following in the form $\sqrt{n}$:

(a) $7\sqrt{2}$ (b) $3\sqrt{3}$ (c) $2\sqrt{7}$

(d) $3\sqrt{7}$ (e) $2\sqrt{2}$ (f) $5\sqrt{5}$

4. Simplify the following as far as possible, leaving your answer in the form $a\sqrt{b}$:

(a) $2\sqrt{3}+5\sqrt{3}$ (b) $7\sqrt{2}-3\sqrt{2}$ (c) $\sqrt{3}+\sqrt{12}$

(d) $\sqrt{27}+2\sqrt{3}$ (e) $5\sqrt{5}+\sqrt{45}$ (f) $7\sqrt{2}+\sqrt{50}$

(g) $\sqrt{18}+\sqrt{200}$ (h) $\sqrt{60}+\sqrt{135}$ (i) $\sqrt{180}-\sqrt{20}$

5. Simplify the following as far as possible:

(a) $2\sqrt{3}\times 5\sqrt{3}$ (b) $5\sqrt{2}\times 2\sqrt{2}$ (c) $\sqrt{3}\times\sqrt{27}$

(d) $\sqrt{50}\times 2\sqrt{2}$ (e) $2\sqrt{7}\times 3\sqrt{28}$ (f) $3\sqrt{3}\times 5\sqrt{75}$

(g) $\dfrac{4\sqrt{2}}{\sqrt{8}}$ (h) $\dfrac{12\sqrt{3}}{\sqrt{48}}$ (i) $\dfrac{10\sqrt{6}}{\sqrt{150}}$

(j) $\dfrac{\sqrt{12}}{\sqrt{300}}$ (k) $\dfrac{3\sqrt{28}}{\sqrt{7}}$ (l) $\dfrac{\sqrt{8}+\sqrt{12}}{\sqrt{2}+\sqrt{3}}$

Histograms

1. (a) Copy and complete the following table which shows the time taken by a group of candidates to finish an exam:

Time in minutes	$45 \le x < 50$	$50 \le x < 55$	$55 \le x < 60$	$60 \le x < 70$	$70 \le x < 90$
Number of candidates	21	32	37	29	18
Frequency Density	$\frac{21}{5} = 4.2$				

(b) Draw a histogram to illustrate this data, using a scale of 1cm per 5 minutes on the horizontal axis (TIME - which goes from 45 to 90) and 1cm per unit on the vertical axis (FREQUENCY DENSITY - which goes from 0 to 8). Label your axes clearly.

2. The height in cm of plants is shown below:

Height (cm)	Frequency	Class Width	Frequency Density
5-	6	5	$\frac{6}{5} = 1.2$
10-	9		
15-	11		
20-	6	10	$\frac{6}{10} = 0.6$
30-	4		
40-60	2		

(a) Copy and complete the above table.
(b) Draw a histogram to illustrate this data, using a scale of 1cm per 5 units on the horizontal axis (which goes from 5cm to 60cm) and 5cm per unit on the vertical axis (which goes from 0 to 2.5).

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3. The table below shows the ages of the people stopped by a company investigating the voting intentions of the adults in a certain town:

Age	18-21	22-25	26-35	36-49	50-74
Age (inequality)	$18 \le a < 22$				$50 \le a < 75$
Frequency	14	24	39	28	20
Frequency Density	3.5				

(a) Explain why the 18-21 category is represented by the inequality $18 \le a < 22$.
(b) Copy and complete the table shown above.
(c) Draw a histogram to illustrate this data, using a scale of 1cm per 5 units on the horizontal axis (which goes from 15 to 75) and 2cm per unit on the vertical axis (which goes from 0 to 6).